University of
St Andrews

HMO CAPS IN ST ANDREWS PROGRESS REPORT 2

CANDLEMAS SEMESTER 2021/22

by

Jen Alexander, Oiza Apeji, Rowan Gibb, Parth Goyal, Bhumi Kanabar, Aniket Khurana, Gracie Kosco, Kieran Pirie, Dhruv Shah, Harry Street, Essia Taj, Grace Tam

Submitted 10 April 2022

Word Count: 6918

Abstract

In 2019, the Fife Council voted to freeze the HMO Licence Cap, restricting the supply of housing available to students looking for private rental accommodation. This semester, the project has aimed to build on the existing research into whether this freeze has impacted the monthly rent prices for students based at the University of St Andrews. By dividing students into four groups based on their income and degree level, the report shows that increased rent prices (due to limited housing supply) limit the number of students that choose to live inside St Andrews. Consequently, students, particularly at the postgraduate level, choose to live in neighbouring towns and cities, such as Guardbridge, Leuchars, and Dundee.

Table of Contents

We would like to express our sincere gratitude to our supervisors, Luc Bridet and Gosia Mitka, for their invaluable support and oversight this semester.

I. Introduction

The aim of the 'HMO Caps in St Andrews' Vertically Integrated Project is to evaluate the economic impact of the HMO licence freeze in the town of St Andrews. Using quantitative econometric modelling, we seek to model the St Andrews housing market to analyse the impact of this policy decision.

Last semester, the team established the foundations of our model that focuses on the decision of individuals to reside within or outside of St Andrews. A binary logit model was introduced and variables of interest that influence the residential choice decision of individuals were established. Further information regarding the progress made last semester can be found in Goyal et al. (2021). This semester has seen an advancement of the model to incorporate a market clearing condition – allowing the team to ask counterfactuals and analyse the impact of the HMO licence freeze on the rental market. There has also been a change in the dependent variables of interest; last semester the model considered the residential choice between 'high' and 'low'-quality housing within St Andrews. The model is now focused on describing the residential choice decision to live either within or outside of St Andrews.

By creating our model in Python, the team finds that there is significant variation in the willingness-to-pay for different groups of students. Based on our artificial data set, the model finds that undergraduates from a high-income background are willing to pay £1036.78, in contrast to £542.33 for low-income undergraduate students. A similar, but smaller, difference is seen for postgraduate students. From these data, the model finds that equilibrium rent is £579.83 inside St Andrews.

The remainder of this paper is as follows. Section II outlines the primary academic papers that have informed our decisions and modelling approach over the semester. This is followed by Section III, which discusses the use of artificial data to test the model's functionality in the absence of survey data. Section IV highlights the econometric theory underlying the model and then proceeds to explain its practical implementation in Python.

Section V presents the findings from our model, which is then discussed in detail in Section VI. Lastly, the next steps are explained in Section VII.

II. Literature Review

A. An empirical evaluation of the probabilistic bid-rent model: The case of homogenous households

Gross (1990) provides a critical analysis of the probabilistic bid-rent model, which explores the unresolved issue in housing economics of the estimation of demand for housing attributes. This paper finds significant drawbacks of the model emphasising that the approach requires the separation of households into homogenous groups, which can be significantly difficult to arrange and consequently can distort estimates of housing preferences. Through this work, we were able to understand why the bid-rent model wasn't the most effective strategy for our project as separating households into homogenous groups is made significantly harder within the student demographic of St Andrews. We hence incorporated these findings to discount this option and instead used probabilistic choice founded on a binary logit model which makes no assumptions regarding homogeneity offering a more simplistic strategy for implementation.

B. A Comparison of binary Logit and Probit models with a simulation study

Cakmakyapan and Goktas (2013) initially recognise the similarities between the Logit and Probit models illustrating how they have the same purposes when the response variable is binary. Using simulation methods, the study goes on to critically evaluate both models under different circumstances measuring the effectiveness of the models by measuring residuals, deviations, and different Pseudo R squares to contribute to effective quantitative analysis. Findings demonstrate that although these models are very similar the Logit Model is better than the Probit Model in larger sample sizes and the probability equation for the Logit Model is easier to model than the Probit Model. As such, we primarily decided to use their methodology in building a binomial logit model.

C. A comparison between Normal and Gumbel Distribution

This report ponders upon the difference between the Normal and Gumbel distributions, despite the main similarity of the appearance of the distributions. The Normal distribution describes a group of continuous probability distributions, whereas the Gumbel distribution models the distribution of the minimum or maximum of several various distributions. When the sample size of the variables of interest reaches larger numbers such as 50, the Gumbel distribution's right-hand side tail becomes heavier, making it useful for predicting the rarer instances within distributions (Qaffou and Zoglat, 2017). Considering our assumptions that St Andrews follows a similar income distribution pattern to the UK population (as discussed below), the Gumbel distribution would have been more useful than a Normal distribution.

D. Deciding on studentification

Our progress report initially researched vertical studentification, later labelling it as less useful to our model. Although a paper by Garmendia et al. (2012) discusses studentification in a small, compact city, it is not material of direct comparison. Notably, it discusses how students end up residing permanently in their university town in the years following university, which seems not to be the case in St Andrews. One conclusion of interest was that studentification occurs in high rises in a hidden way, with vertical studentification being prominent over its horizontal counterpart. The only way this links into our model, is that vertical studentification is more likely to occur living outside of St Andrews, and horizontal studentification is most likely to occur living inside of St Andrews.

E. The Tarbase Domestic Model

The Tarbase (Technology Assessment for Radically improving the Built Asset base) Domestic Model (Jenkins et al., 2011) is used to model carbon emissions, and was first developed to evaluate measures to potentially be implemented for carbon-saving refurbishments, can be added to our model depicting the St Andrews housing market. The model is effective, as it allows for both the internal activity of a particular house, as well as its external environment, to be incorporated. It uses various algorithms through a simple spreadsheet framework, subsequently providing options for carbon-saving targets. Given the distinctive elements of an HMO, this model turned out to be less useful, and we instead hoped to elucidate the true effects of carbon emissions from commuting, seeing it as more apt (see section F below). Thus, the Tarbase Domestic Model is of the remit of this year's VIP cohort due to its peculiarities, as well as due to time constraints, but remains a valid part of the literature review and could be researched further by next semester's cohort.

F. Reducing the number of commuters to St Andrews

This semester, we tried to determine whether more students living in town (and thus closer to university), would reduce the CO2 emissions caused by their commuting. A paper by Filimonau, et al. (2021) discusses the influence of the lockdown - which decreased the number of commuters - on CO2 emissions. We consider that students who live within a 15-30 min walking distance from the University, will walk to class or use a bike, rather than driving or using public transport. The model we are investigating could help us elucidate the effects the HMO cap has had on commuting students to St Andrews, hence the contribution to carbon emissions. This model is also relevant in formalising this understanding in models that describes the current effect of commuting and the pattern of energy. It produces tools to assist those who can make a difference to reduce CO2 emission by allowing more students to be able to not consider commuting by car or bus.

III. Data

The present discrete choice model differentiates between inside and outside of St Andrews. The input variables used in the model and, therefore, required for the dataset are the following:

- Average rent price
- Dummy variable for degree level
- Dummy variable for income level

Income Level for Individuals

Income level is a binary variable, where 1 = High Income; 0 = Low Income. To determine the two groups, we used data from a survey conducted by the Office for National Statistics (ONS) in 2020. It found that mean income in the UK was £36,900, and 64.6% of the population sit below this mean. Therefore, the probability that an individual is from a High-Income family is 0.354.

Degree Level

Degree level is the other binary input variable, where 0 = Postgraduate; 1 = Undergraduate. The probability that an individual is a postgraduate is 0.208, which is based on data from the University, published for this academic year. 2,164 of the 10,425 are postgraduate students.

We assume that degree level and income level are independent of each other; if a student is an undergraduate, they are not more or less likely to be from a low/high-income family.

Student Housing Choice

The model takes house prices as binary, where there is a price for living in St Andrews, and one price for living outside of St Andrews in neighbouring towns and cities. We calculated these rent prices from a 2013 report, conducted for the University of St Andrews and Fife Council (MacLennan et al., 2013). This report found that the average rent price inside and outside of St Andrews was £440 and £398, respectively.

Based on rent increases from 2010-2021 in Fife and Dundee, we are able to calculate current estimates of average rent prices for St Andrews and outside St Andrews, respectively. Thus, this gives us:

£584.32 (current estimate for Inside St Andrews)

£480.00 (current estimate for Outside St Andrews)

To decide whether a student chooses to live inside or outside of St Andrews depends on their degree level and family income. Therefore, we can divide our students into four groups. A given student will fall into one of the four below categories:

1. High Income | Undergraduate
2. Low Income | Undergraduate
3. High Income | Postgraduate
4. Low Income | Postgraduate

If a student is from a high-income family and is an undergraduate student, then the probability they live in St Andrews is equal to 0.8, whilst low-income undergraduates have a probability of 0.6. We assume that undergraduate students derive high utility from living inside of St Andrews, because undergraduate students have more contact hours and are more involved with Student's Union clubs and societies. Therefore, even for those with low-income, we expect they are willing to pay the rent premium to live in St Andrews; subsequently, a large proportion of undergraduates will live in town.

For postgraduate students, we estimate that the probability for high-income postgraduates to live in St Andrews is 0.7, whereas it is 0.5 for those from a low-income background. Data suggests that postgraduates have a higher probability of living outside of St Andrews. Furthermore, the higher rent price inside St Andrews would likely drive out low-income postgraduates from living inside of St Andrews, as their willingness to pay in the St Andrews market is lower compared to undergraduate students.

Variable	**Excel Function**
Rent	=IF(D2="1"&E2="1",(IF(RAND()<=0.8,"584.32","480")),IF(D2="0"&E2="0",(IF(RAND()<=0.5,"584.32","480")),IF(D2="1"&E2="0",(IF(RAND()<=0.7,"584.32","480")),IF(RAND()<=0.6,"584.32","480"))))
High Income = 1	=IF(RAND()<=0.354,"1","0")
Undergraduate = 1	=IF(RAND()<=0.792,"1","0")

Table 1: Excel functions to create artificial data set based on variables of rent, income, and degree level.

IV. Empirical Strategy

A. Theoretical Model

The model describes a residential choice decision where student x must choose between two exclusive, well-defined, and exhaustive options – living inside of St Andrews, or living outside of St Andrews. Using a binary logit model and random utility theory as described in McFadden (1978), we establish the probability of a student choosing to live within St Andrews.

As discussed in the data section, students are categorized into four discrete bins:

1. Low-income, undergraduate
2. Low-income, postgraduate
3. High-income, undergraduate
4. High-income, postgraduate

The variables of interest relating to student x's decision to reside within or outside of St Andrews are given by:

- Family income (£)
- Level of study (undergraduate/postgraduate)
- Average rent inside St Andrews (£)

Each student selects the residential choice option that maximises their utility, which depends on observable and unobservable characteristics of both their idiosyncratic preferences and the residential choice option itself.

The utility maximization function of student x derived from housing choice i is given by:

$$\max_{x} U_x^i = aI_x - bR_i + cD_x + \varepsilon_{xi} \tag{1.1}$$

where;

a, b, c are coefficient parameters

I is a binary variable of family income

R is a binary variable of the monthly rent of the property option

D is a binary variable of the student's degree level

ε_{xi} is an error term

The error term, ε_{xi}, accounts for variation in data and unobserved characteristics that influence an individual's choice.

Assuming a logistic distribution of ε, the probability $P_x^{i(inside)}$ of student x choosing to live within St Andrews, $i(inside)$, is given by:

$$P_x^{i(inside)} = f(I_x, R_i, D_x, \varepsilon_{xi}) \tag{1.2}$$

Aggregating the probabilities in equation (1.2) over all students yields the predicted number of students that choose to live inside St Andrews, $N_x^{i(inside)}$:

$$N_x^{i(inside)} = \sum_x P_x^{i(inside)}$$

(1.3)

This is an application of the law of large numbers. We assume that by aggregating the individual probabilities, the overall average will converge toward the theoretical mean. We can expect the observed value to converge with the expected value.

The assumption is made that the supply of properties within St Andrews is fixed as described in the 'data' section.

The market-clearing quantity for properties within St Andrews is attained by solving a system of equations. The rent at which the market clears, written as $R^*_{i(inside)}$, is at the point where the demand for St Andrews properties, $N_x^{i(inside)}$, equals the pre-defined supply and can be expressed as:

$$R^*_{i(inside)} = R \ni St\ Andrews\ Property\ Supply = N_x^{i(inside)}$$

(1.4)

$R^*_{i(inside)}$ is obtained by marginally increasing the independent rent variable (R_i) within the probabilistic choice formula (1.2) until the condition described in (1.4) is satisfied.

The demand curve of each student group for properties within St Andrews is calculated by plotting the change in $N_x^{i(inside)}$ as R_i is increased.

$$Demand^x_{i(inside)} = P_x^{i(inside)} * N_x$$

(1.5)

where N_x is the number of students in group x

The total demand for properties within St Andrews is given by:

$$Total\ Demand = \sum_{x}^{i(inside)} N_x^i = N_{x(UG,low)}^{i(inside)} + N_{x(UG,high)}^{i(inside)} + N_{x(PG,low)}^{i(inside)} + N_{x(PG,high)}^{i(inside)} \tag{1.6}$$

B. Coding

The following is a step-by-step run-through of the full model in Python. For reference, the full code is available in Appendix 1.

The code required several Python packages and libraries to be imported. Python packages contain a collection of related modules and packages whereas a library contains both modules and packages. In the following code we have used the packages displayed in Table 2.

Package Name	**Description**
NumPy	NumPy performs a wide variety of mathematical operations on arrays.
SciPy Optimize	SciPy Optimize is useful for performing a wide variety of mathematical operations on arrays and many commonly used optimization algorithms. In particular, the importation of the Minimize application is central to the calibration methodology.
matplotlib.pyplot	This interface is used to plot outputs.
math	Provides access to mathematical functions.
Pandas	A Python library that is useful for data analysis and manipulation.

Table 2: Enumeration and description of Python packages.

The code solves for eight unknowns. These unknowns are shown in Table 3. The code incorporates these unknowns into the functions outlined in the theoretical model. In this way, it creates a system of equations. To solve for this system, we created other independent moments that include these unknowns and run a code which finds a solution for the system to minimize the error. This is the calibration methodology, used to find the equilibrium values of the unknowns using SciPy Optimize Minimize. The steps to this calibration methodology are to create the functions with the unknowns, set objectives for those parameters using independent theoretical moments, set boundary conditions on the possible outcomes, constrain willingness to pay to be higher for high-income students, and to find the solution with the smallest possible error using SciPy Optimize Minimize.

Unknown Parameters	**Name**
Mean willingness to pay for undergraduates who are low income	WTP_UGL
Mean willingness to pay for undergraduates who are high income	WTP_UGH
Mean willingness to pay for postgraduates who are low income	WTP_PGL
Mean willingness to pay for postgraduates who are high income	WTP_PGH
Proportion of undergraduates who are low income	Prop_UGL
Proportion of postgraduates who are low income	Prop_PGL
Variance of the error term	sig
Equilibrium rent	R

Table 3: Unknown parameters for which the code solves.

A series of constants are utilized in the model. The disutility of not being in St Andrews is set to zero. This number is arbitrary and is only important in relation to the utility of being in St Andrews. The utility of being in St Andrews in this model, one of the unknown parameters, is the mean willingness to pay for each student group. The number of undergraduate and postgraduate students matriculated into the University of St Andrews was found to be 8260 and 2164, respectively. Finally, the number of rooms available in HMO properties was found to be 7156.

The piece of code below is used to define the PSA function, which computes the sigmoid curve of the utility function of not living in St Andrews. This is subtracted from one to find the probability of being in St Andrews dependent on the utility.

In the code below, `i` stands in for the student group labels. In Python, these functions are repeated for each student group. Therefore, this `PSA` function will be nested in each student group's individual demand function.

```
def PSA(utility):
       return  (1-1 / (1 + e**((utility - Disutility_notSTA)/sig)))
```

Each student group has a demand function based on the variables of the proportion of students in either degree level who are low-income, the willingness to pay of the student group, the variance of the error term, and equilibrium rent. Below shows an example of this demand function. In the final code, there are four versions of the function, one for each student group.

```
def demand_bin(Prop_degreelevel_L,WTP_bin,sig,R):
    bin = Prop_degreelevel_L*Num_degreelevel*(PSA(WTP_bin,sig,R))
    return bin
```

Subsequently, the functions with the unknowns are set. The next part is about using theoretical moments formed to solve for the various parameters. `objective` is the function that holds these theoretical moments. SciPy Optimize Minimize will use them as goals, but they can change in order to find a solution to the system of equations. However, they will change in such a way that the error, or difference between the objective and the solved model, is minimized. Note that SciPy Optimize Minimize sets these equal to zero. Therefore, any constants that are non-zero are added or subtracted to make the objective equal to zero.

```
def objective(parameters):
```

```
(Prop_UGL,Prop_PGL, WTP_UGL, WTP_UGH, WTP_PGL, WTP_PGH, R,sig) =
parameters
```

The inputs to the `objective` function are the unknown parameters from before. The first objective is that of elasticity. We assume that the elasticity of demand is close to -0.76. This means that for each additional unit of `R`, that is, for each pound added onto equilibrium rent, the demand will decrease by 0.76 percent. In the following lines of code, `total_demand` finds the quantity demanded at equilibrium rent. Then, `total_demand_2` finds the quantity demanded if `R` increases by one unit. Lastly, `elasticity_D` says that the percentage change from `total_demand` to `total_demand_2` plus 0.76 is equal to zero.

```
elasticity_D = (total_demand_2-total_demand)*R/(total_demand) + 0.76
```

The next objectives are for the proportion of each student group living in St Andrews. The objective proportions are 0.6, 0.8, 0.5, and 0.7 for undergraduate low income, undergraduate high income, postgraduate low income, and postgraduate high income, respectively.

```
Proportion_UGL =
demand_UGL(Prop_UGL,WTP_UGL,sig,R)/(Prop_UGL*Num_UG) - 0.6

Proportion_UGH = demand_UGH(Prop_UGL,WTP_UGH,sig,R)/((1-
Prop_UGL)*Num_UG) - 0.8

Proportion_PGL =
demand_PGL(Prop_PGL,WTP_UGH,sig,R)/(Prop_PGL*Num_PG) - 0.5

Proportion_PGH = demand_PGH(Prop_PGL,WTP_PGH,sig,R)/((1-
Prop_PGL)*Num_PG) - 0.7
```

Continuing in this way, `objective` states that equilibrium rent should be near £580 and the market clears when total demand is equal to total supply. `error_sum` subtracts 0.5 from the proportions of the degree level that are low income because the proportions should not significantly deviate from 0.5. In this way, there is a penalty for the solution having extreme values for these proportions. It is then squared to make certain they are positive, as their definition necessitates. `P_c` will not change the solution. When changed, it is used to simplify the sensitivity analysis procedure.

```
Rent = R - 580
Market_clearing = total_demand-Quantity
P_c = 1 # constant for proportion
error_sum = P_c*((Prop_UGL-0.5)**2 + (Prop_PGL-0.5)**2)
```

Then, all these objectives are gathered up into `list_of_equations`. `Market_clearing` is divided by 7156 to make sure that no parameter error has an outsized impact. `Market_clearing` is compared to proportions between zero and one, so dividing it by 7156, we remove that unequal impact. The same is true for `Rent`. They are both put to the power of 0.5 because they will be squared later. `P_c`, `E_c`, `M_c`, and `R_c` all currently do not change the outcome but are used to perform sensitivity analysis.

```
list_of_equations =
[Proportion_UGL,Proportion_UGH,Proportion_PGL,Proportion_PGH,Market_clearing/7156**0.5,elasticity_D, Rent/580**0.5]
```

The code below runs a loop for all the objectives in `list_of_equations`. SciPy Optimize Minimize finds the solution closest to zero. Therefore, `i` is the nominal error for each objective. By squaring this number, it is certainly positive. After summing all the positive errors, SciPy Optimize Minimize will minimize this cumulative error term.

```
for i in list_of_equations:
    error_sum = error_sum + i**2
    return error_sum# returning the list of equations
```

The next step in the calibration process is to set hard boundary conditions on the possible outcomes. These limits are set in a list which corresponds to the list of parameters from the definition of `objective`. Therefore, `Prop_UGL` and `Prop_PGL` are constrained by `prop_limit`, the four willingness to pay variables are constrained by `WTP_limit`, `R` is constrained by `rent_limit`, and `sig` is constrained by `sig_limit`.

```
prop_limit = (0.02,0.98)
WTP_limit = (300,1500)
rent_limit = (350,850)
sig_limit = (2,1000)
limits = (prop_limit,prop_limit,WTP_limit,WTP_limit,WTP_limit,WTP_limit,rent_limit,sig_limit)
```

Another necessary constraint is that willingness to pay must be higher for high-income students than for low-income students. `constraint_UG` finds the difference between undergraduate high- and low-income students' willingness to pay and `constraint_PG` does the same for postgraduate students. Then `con1` and `con2` are inequalities such that `WTP_UGH` must be greater than or equal to `WTP_UGL` and `WTP_PGH` must be greater than or equal to `WTP_PGL`. Then, `cons` are run through the SciPy Optimize Minimize calculation.

```
def constraint_UG(parameters):    #inequality
    (Prop_UGL,Prop_PGL, WTP_UGL, WTP_UGH, WTP_PGL, WTP_PGH, R,sig) = parameters
    return WTP_UGH-WTP_UGL

def constraint_PG(parameters):   #inequality
    (Prop_UGL,Prop_PGL, WTP_UGL, WTP_UGH, WTP_PGL, WTP_PGH, R,sig) = parameters
    return WTP_PGH-WTP_PGL
con1 = {'type':'ineq','fun':constraint_UG}

con2 = {'type':'ineq','fun':constraint_PG}

cons = [con1,con2]
```

With all the information coded above, SciPy Optimize Minimize can solve the system. `Guess` is used to help point Minimize in the correct direction. Finally, `minimize` uses the objectives, the `Guess`, the method SLSQP (Sequential Least Squares Programming), the hard coded bounds, and the constraints on willingness to pay to find `Results`.

```
Guess = [0.1,0.2,500,550,525,575,20,500]

Results = minimize(objective,Guess,method='SLSQP',
bounds=limits,constraints=cons)
```

V. Results

A. Main Findings

Using the findings from `Results`, we can find equilibrium demand for each student group by plugging the results back into the `demand_bin` function, elasticity from `elasticity_D`, and the relevant proportions. This outputs the data frame shown in Figure 1. The market equilibrium is displayed in Figure 2.

	UGL	UGH	PGL	PGH	Total
Proportion of Students Who are Low/High Income	0.351965	0.648035	0.482863	0.517137	N/A
Proportion of Students Living in St. Andrews for Each Bin	0.4615	0.8675	0.4676	0.6091	N/A
Proportion of St Andrews Rooms Let to Each Bin (Demand of Bin/Total Rooms)	0.1875	0.6489	0.0683	0.0953	N/A
Willingness to Pay for each Bin & Equilibrium Rent	542.33	1036.78	548.31	687.66	579.83
Number of Students from each Bin Living in St Andrews	1342.0	4644.0	489.0	682.0	7156.0
Elasticity of Demand	N/A	N/A	N/A	N/A	-0.621536

Figure 1: Data frame output from the calibrated model.

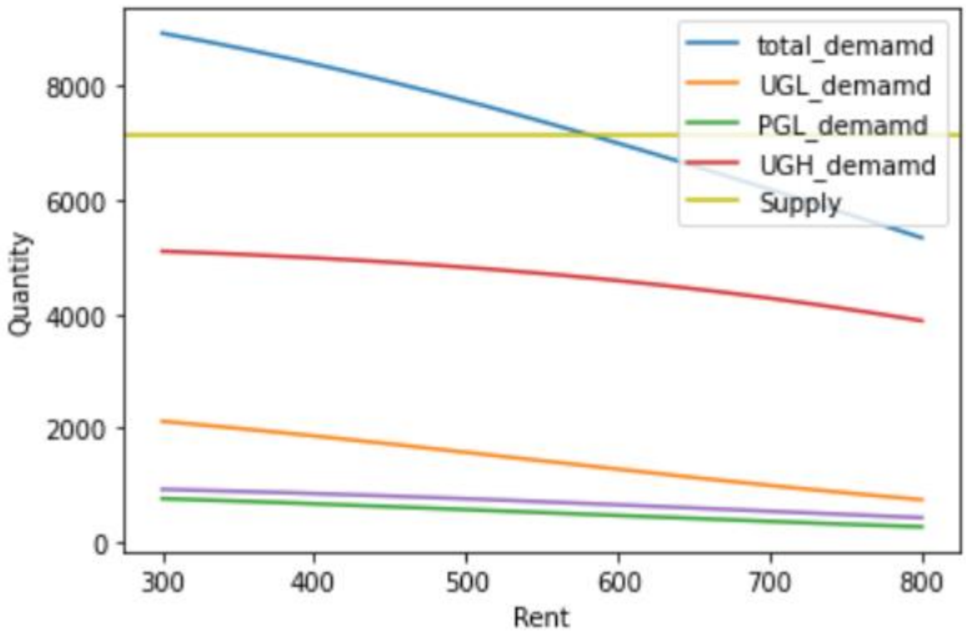


Figure 2: Graphical output of solution. Intersection of Supply and total_demand is market equilibrium.

B. Sensitivity Analysis

We define sensitivity analysis as the analysis of observed change in our parameter values during calibration based on a change in the associated weight of our objectives.

Weights are assigned to the significant objectives of our model using the scalar constants as defined below

```
P_c = 1 # constant for proportion
E_c = 1 # constant for elasticity
M_c = 1 # constant for market clearing
R_c = 1 # constant for Rent#
```

Null state is identified as equal weight across all objectives; hence all constants are equal to 1. This state is depicted in Figure 1.

The methodology is changing the constants one by one, while keeping others constant and analysing the nominal deviation of the parameters from the null state. Various scenarios have been considered and reported in an excel sheet (Appendix 2). Only significant findings will be discussed here.

P_c increased to 2

Change in which parameter	*UGL*	*UGH*	*PGL*	*PGH*	*Total*
Willingness to Pay for each Bin & Equilibrium Rent	262.75	-231.7	43.49	102.3	0.24
Proportion of Students Who are Low/High Income	0.147	-0.147	0.0138	-0.0138	
Proportion of Students Living in St. Andrews for Each Bin	0.2465	-0.1595	0.0439	0.0865	
Proportion of St Andrews Rooms Let to Each Bin (Demand of Bin/Total Rooms)	0.2199	-0.239	0.0085	0.0106	
Number of Students from each Bin Living in St Andrews	1573	-1711	61	76	0
Elasticity of Demand					-0.0832

E_c increased to 2

Change in which parameter	UGL	UGH	PGL	PGH	Total
Willingness to Pay for each Bin & Equilibrium Rent	253.29	-241.16	37.95	38.99	0.15
Proportion of Students Who are Low/High Income	0.1582	-0.1582	0.0101	-0.0101	
Proportion of Students Living in St. Andrews for Each Bin	0.2529	-0.1531	0.0391	0.0419	
Proportion of St Andrews Rooms Let to Each Bin (Demand of Bin/Total Rooms)	0.2332	-0.245	0.0072	0.0045	
Number of Students from each Bin Living in St Andrews	1669	-1754	52	32	0
Elasticity of Demand					-0.1376

M_c increased to 2

Change in which parameter	UGL	UGH	PGL	PGH	Total
Willingness to Pay for each Bin & Equilibrium Rent	252.38	-242.07	32.29	104.97	0.2
Proportion of Students Who are Low/High Income	0.1788	-0.1788	0.008	-0.008	
Proportion of Students Living in St. Andrews for Each Bin	0.2463	-0.1597	0.033	0.0969	
Proportion of St Andrews Rooms Let to Each Bin (Demand of Bin/Total Rooms)	0.2462	-0.2656	0.006	0.0133	
Number of Students from each Bin Living in St Andrews	1761	-1901	43	95	0
Elasticity of Demand					-0.1148

R_c increased to 2

Change in which parameter	UGL	UGH	PGL	PGH	Total
Willingness to Pay for each Bin & Equilibrium Rent	-25.93	43.96	-17.67	25.94	0.14
Proportion of Students Who are Low/High Income	-0.0263	0.0263	-0.0058	0.005776	
Proportion of Students Living in St. Andrews for Each Bin	-0.0191	-0.0067	-0.0123	0.0101	
Proportion of St Andrews Rooms Let to Each Bin (Demand of Bin/Total Rooms)	-0.0212	0.0212	-0.0026	0.0026	
Number of Students from each Bin Living in St Andrews	-152	151	-19	19	-1
Elasticity of Demand					0.0746

Table 4,5,6,7: Deviations from Null State under Different Circumstances.

Only increases in the constants have been reported as the results from decreases did not show significant deviations from the null state.

Proportion and WTP parameters are sensitive when M_c, E_c and P_c is increased, especially for Undergraduate groups. Along with that, there seems to be some deviation in elasticity as well, suggesting some uncertainty in our elasticity estimate.

On the other hand, our rent and total supply estimates are very insensitive, as there is almost no deviation from the null state.

These results are representative of our data accuracy and theoretical moments. We have up to date and detailed data on rent and HMO supply in St Andrews, whereas our proportion estimates are arbitrary and based on a synthetic data set and elasticity is approximated based on general literature.

Another possible explanation for these results is that the weights in our null state are not accurate or standardised appropriately, resulting in such a difference between our parameters.

C. Counterfactual Analysis

We define counterfactual analysis as the impact of significant shocks to the current null state. The null state is defined similarly as before; however, this time rent is variable as it's the primary parameter of interest. So we are solving for rent using Fsolve while keeping all other parameters constant .

The methodology is introducing an external shock to the null state, such as increasing housing supply while keeping everything else constant and observing the change in parameters of interest: Proportion and Number of Students in St Andrews for each Bin, Elasticity and Equilibrium Rent.

Increasing Housing Supply by 3%

Change in which parameter	UGL	UGH	PGL	PGH	Total
Proportion of Students Living in St. Andrews for Each Bin	0.0292	0.0129	0.0293	0.0275	
Proportion of St Andrews Rooms Let to Each Bin (Demand of Bin/Total Rooms)	0.0061	-0.0095	0.0021	0.0013	
Number of Students from each Bin Living in St Andrews	85	69	30	31	215
Elasticity of Demand	0.06435352458				
Equilibrium Rent	-28.47				

At the time of implementing the HMO licence freeze, the council considered an alternative increase in HMO licences of 3%. Therefore, we believe that 3% is a reasonable annual HMO licence growth figure in the absence of the freeze. As expected, our model predicts a modest increase in the number of students in each bin living inside St Andrews, and a fall in equilibrium rent.

Increasing number of students to 11000. UG(+458) PG(+116)

Change in which parameter	UGL	UGH	PGL	PGH	Total
Proportion of Students Living in St. Andrews for Each Bin	-0.0482	-0.0242	-0.0483	-0.0476	
Proportion of St Andrews Rooms Let to Each Bin (Demand of Bin/Total Rooms)	-0.0103	0.0169	-0.0038	-0.0028	
Number of Students from each Bin Living in St Andrews	-74	120	-27	-19	0
Elasticity of Demand	-0.1176311454				
Equilibrium Rent	+47.7				

We expect the University to reach a population of 11,000 students by the 2023/24 academic year, based on a 2.71% CAGR. This aligns with the historic University growth rate over the past 10 years of 2.70%. Our model predicts a significant increase in equilibrium rent and a redistribution of students living inside St Andrews into the undergraduate high-income bin from all other bins.

Decreasing number of students to 10000. UG(-340) PG(-86)

Change in which parameter	UGL	UGH	PGL	PGH	Total
Proportion of Students Living in St. Andrews for Each Bin	0.0418	0.0181	0.0419	0.039	
Proportion of St Andrews Rooms Let to Each Bin (Demand of Bin/Total Rooms)	0.0086	-0.0137	0.0031	0.002	
Number of Students from each Bin Living in St Andrews	61	-99	22	16	0
Elasticity of Demand	0.09065926458				
Equilibrium Rent	-40.73				

Prior to the COVID-19 pandemic, the University's strategic plan indicated an ambition to grow the University to 10,000 students by 2025. We have therefore analysed the impact of reducing the student population to this target figure.

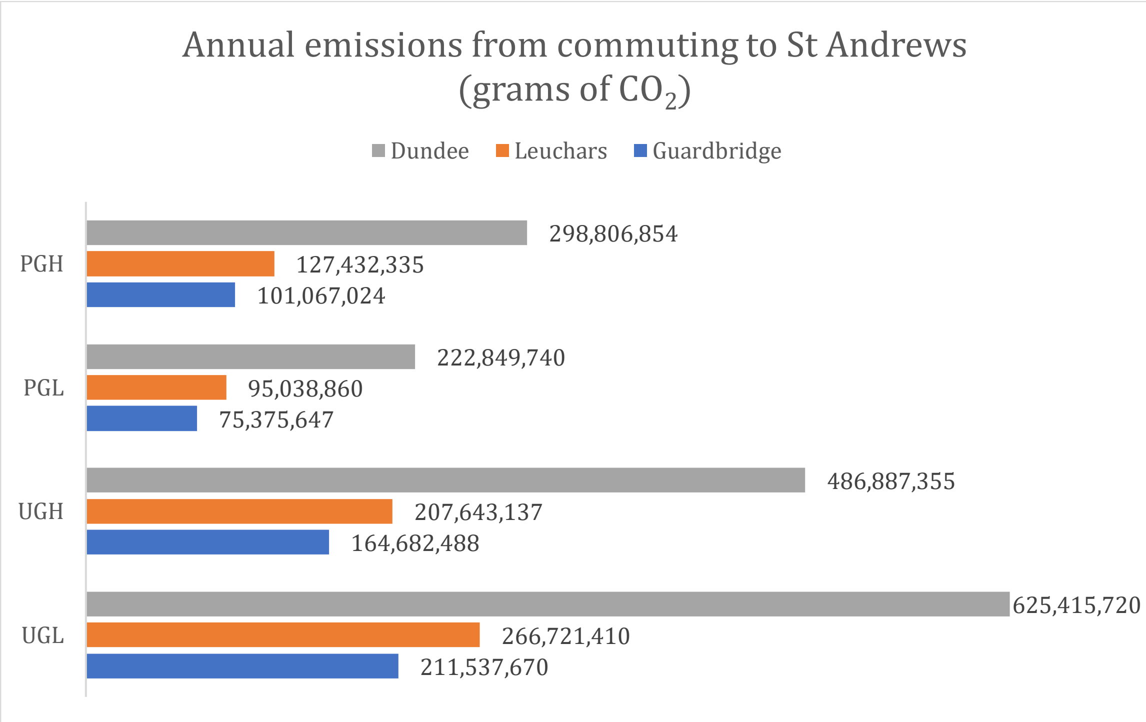
Introducing (- 100) units as Utility of not being in St Andrews

Equilibrium Rent	99.4

Table 8,9,10,11: Deviations from Null State under different Shocks

D. Environmental Impacts of Commuting

To extend our research, we estimated the greenhouse gas emissions produced when students commute to St Andrews.



The number of students living outside of St Andrews was taken from the core model, and the distance was taken from Google Maps. We used STATISTA to calculate the carbon emissions per mile per bus.

We assume that students attend approximately three classes per week (both postgraduate and undergraduate), requiring a return journey to St Andrews. We assume that a student will take a 90 annual round trips. We further assume that low-income students will take the bus and high-income students will drive to St Andrews.

The formula below illustrates an example of how emissions were calculated for a low-income undergraduate student living in Guardbridge.

$$\begin{aligned} &annual\ emissions\ UGL \\ &\qquad = no.of\ UGL\ living\ in\ Guardbridge \\ &\qquad * distance\ between\ St\ Andrews\ and\ Guardbridge \\ &\qquad * carbon\ emissions\ per\ mile\ per\ bus * 90 \end{aligned}$$

The graph above depicts the annual carbon emissions from commuting students. The presence of the HMO licence cap likely increases the number of students living outside of St Andrews; thus, has a tangible impact on the carbon footprint of the university.

VI. Discussion

A. Data

Our model uses simulated data based on summary statistics that we found through web scraping. Because we use simulated data to represent our interest group, there are some limitations to the validity of our model. One of which is a trade-off between accuracy of the datapoints and time that we would have otherwise taken to survey the population. The simulated data is easier to gather but it is not certain to be accurate because it was not generated from a true population. On the other hand, if we were to run a survey to gather data it is not certain that the sample we get will be representative, as there is always the possibility of sampling bias. As such, it is likely that the use of artificial data would not significantly affect how representative of the population the sample is.

The use of binary data for the income variable also limits the model in its ability to make precise predictions. We can only talk about the outcomes of the HMO supply and demand levels in terms of proportions and counts and we have only a few categories to place students in. Meanwhile in reality the observations would lie on a spectrum (e.g., of low income to high income students). However, this approach is still feasible as we are interested in making predictions about the general population of St Andrews students and the binary data is sufficient for this purpose.

The problem of omitted variable bias is also present here since we are only using three dependant variables. It is highly likely that there are more variables which contribute to the utility of the housing. For example, the amenities in the house or what course the student studies or whether they are an international student. Some of these variables can be correlated with rent and since they are omitted from our utility regression, they result in biased estimated coefficients.

To simplify the dataset, we have made two assumptions about family income. Firstly, we assume that family income is independent of whether a student is either here as an undergraduate or postgraduate student. Secondly, we assume that the St Andrews income distribution is the same as the UK: 35.4% of the population sit above the mean household income. Though these assumptions allowed us to produce a dataset more simply, it is unlikely that either would be true for a real-world data set. Over 45% of students and staff at the university come from outside of the United Kingdom; therefore, it is unlikely that they will all follow the same income distribution seen in this country. Therefore, it is unlikely that the cut-off for high and low-income would sit at the proposed figure.

B. Model

The model is based on a two-step framework. The first is the minimum distance calibration method, which yields our parameter estimates. The second is the implementation of the binary logit model using our parameters. It is a significant improvement from the start of the semester as our parameter estimates are not based on arbitrary numbers anymore and we have established a model that can be effectively calibrated and implemented to perform sensitivity and counterfactual analysis. Using the results, potential policy decisions to mitigate the impacts of the HMO licences can be discussed and tested.

Despite having a simple binary logit model, the current framework gives many important results about the St Andrews housing market, such as various proportions, elasticity of demand and Rent.

However, there are some drawbacks of our current framework. The current calibration method is not entirely representative of real-world due to the limited data about the St Andrews housing market, increasing uncertainty and sensitivity of our estimates of certain parameters with a lingering possibility of inappropriately standardised errors as depicted in the section V.B.

Additionally, we were forced to switch to the use of minimum distance method since there weren't any analytical or numerical solutions to the system of equations formed by our theoretical moments in the objectives along with lack of flexibility caused my limited number of moments.

Our theoretical model's simplicity enforces an assumption that is unreasonable for St Andrews. The assumption of homogenous housing stock. St Andrews has a variety of housing choices with distinct characteristics that have a significant impact on individuals' utility and choice, which is disregarded in the model.

C. Environmental Impact of HMO Licence Cap

Our back-of-the-envelope calculation has significant limitations due to the use of excessively relaxed assumptions and arbitrary numbers. For example, the prospect of multiple students travelling on a single bus is not considered. Additionally, the assumption that all high-income students travel by private car is likely to be an overestimate. Therefore, overall, our emission calculations are likely overestimated. Finally, the model assumes that all vehicles are fuelled by petrol, which fails to consider diesel or electric cars. Future work in this area should seek to strengthen the assumptions made and replace the arbitrary numbers with empirical data.

D. Communicating our work – Inaugural VIP Conference

The first Vertically Integrated Project conference was held on the 22nd March 2022. The event was an excellent opportunity to receive feedback from the public and share our research with the wider academic community. The feedback received was overwhelmingly positive and the setup of our stall was particularly esteemed. The team won the 2nd place award for 'best display'. We also received positive feedback regarding the enthusiasm of the team and our engagement with the research theme.

We endeavoured to make our display interactive and simple to effectively convey our research to a general audience. We used two posters, an interactive map of St Andrews on Java, and miniature houses which were placed on a physical map. The posters contained information on modelling and Python outputs from the two most recent semesters. The interactive demand on Java allowed visitors to observe how their willingness to pay compared to the wider population. The miniature houses placed on a map of St Andrews and the surrounding area created an interactive element which sparked conversations and allowed conference attendees to personally engage with our research. We placed four houses in different locations (student's union, Morrisons supermarket, Guardbridge, and Leuchars). Each house had an attached rent price. Conference attendees were asked to select their preferred housing option. Subsequently, we removed a house in St Andrews to see whether this would affect their choices. This illustrated the impact of the HMO cap supply constraint, and the concept of utility maximisation theory using Layman's terms.

The success of our approach was highlighted by the intelligent questions asked by those with no economic background, implying that we had effectively conveyed our research.

The only issue with understanding regarding the poster was the python output, as this is arguably the most technical part of the poster it likely requires further explanation on the poster. In terms of planning for the conference, there were organisational issues that stemmed from a lack of knowledge about the structure of the event and could be remedied in future by asking for the event program as far in advance as possible. One consequence of this issue was too many team members on hand and without defined roles blocking the stall from view. In future, greater knowledge of the event structure would have allowed us to better prepare for the circumstances and enable us to take full advantage of the excellent opportunity to find out about other VIP projects.

Many innovative and exciting possible next steps for the project were suggested over the course of the conference. One such being interviewing both locals and students on their feelings about the HMO caps. This would enable us to see the impact of our project beyond the university and be an excellent opportunity for student to hear the locals' perspective, potentially beginning to bridge the gap within our community. The idea of speaking to Fife Council about the HMO caps was also raised as a valuable way of allowing the team to see the project from both sides. Other VIP teams reported using interviews in this way to be very successful and took it a step further by hosting in person events to engage with external stakeholders and the wider university community. As housing in St Andrews is a topic many people feel very strongly about, this could be an excellent opportunity to hear more feedback, generate new ideas and incite conversation about HMO caps in St Andrews. Additionally, using QR codes in our poster and working on a social media presence could be an excellent next step to make the project more engaging, these techniques already being used effectively by other teams. Finally, discussion of the UN sustainable development goals at the conference has allowed us to connect the project to climate action by considering the impact of HMO caps on commuting emissions.

VII. Next Steps

A. Data

Moving forwards, an imperative goal is to conduct a survey of the student population. This would allow the project to incorporate real-world data and would not have to rely on web scraping and assumptions, which limit the accuracy of the data used to calibrate the model.

There would be two aspects to the survey. Firstly, we could better understand the average rent prices inside and outside of St Andrews; instead of relying on average rent inflation, the team could gather accurate, recent rent prices that students are currently paying.

Secondly, a survey would enable us to better understand where students of different bins would choose to live and how family income, country of birth, and degree level may affect rental choices. This will enable future teams to estimate where students will choose to live, and the willingness-to-pay for different student groups.

A team dedicated to designing a survey will be needed to ensure that the relevant questions are included to gather sufficient data. In addition, the data team will need to ensure the necessary procedures are followed in a timely manner, so the survey can be conducted on behalf of the university within the limited timeframe.

B. Model

Following from the discussion section, there are several areas the modelling team can improve on.

1. Doing a more comprehensive sensitivity analysis by considering the rate of change for several parameters using partial derivatives. This will allow for a more standardised approach and give better insights into the calibration process.

2. Generation of more independent theoretical moments so it increases flexibility in the calibration process, reducing uncertainty and increasing the accuracy of estimates.
3. Extending from a binary logit model to a multinomial one, accounting for the heterogenous housing stock and increasing the results the models predict, so more informed policy decisions can be made.

References

General Citations

Bayer, P., McMillan, R. and Rueben, K., 2004. An equilibrium model of sorting in an urban housing market. National Bureau of Economic Research Working Paper No. w10865. Available at: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=612067 (Accessed 2 March 2022)

Goyal, P., Khurana, A., Pirie, K. and Street, H., 2021. HMO Caps in St Andrews Progress Report – Martinmas Semester 2021/22.

MacLennan, D., Maynard, K., O'Sullivan, T., Sila-Nowicka, K. and Walden, Y., 2013. Growing Economies and Building Homes: Reconciling Growth and Housing Wellbeing in St Andrews.

McFadden, D., 1978. Modelling the choice of residential location. Transportation Research Record, 673(1), pp.72-77.

Literature Review

Cakmakyapan, S. & Goktas, A., 2013. A comparison of binary logit and probit models with a simulation study. Journal of Social Economic statistics, 2(1), pp. 1-17.

Garmendia, M., Coronado, J. M. & Ureña, J. M., 2012. University students sharing flats: When studentification becomes vertical. Urban Studies, 49(12), pp. 2651-2668.

Gross, D. J., Sirmans, C. F. & Benjamin, J. D., 1990. An empirical evaluation of the probabilistic bid-rent model: The case of homogenous households. Regional Science and Urban Economics, 20(1), pp. 103-110.

Jenkins, D. P. et al., 2012. Modelling carbon emissions of UK dwellings – The Tarbase Domestic Model. Applied Energy, Volume 93, pp. 596-605.

Qaffou, A. & Zoglat, A., 2017. Discriminating between normal and gumbel distributions. REVSTAT-Statistical Journal, 15(4), pp. 523-236.

Appendix 1

```
import matplotlib.pyplot as plt

import numpy as np

import math

from scipy.optimize import minimize

import pandas as pd


# ^^^ these are the relevant libraries and packages


Disutility_notSTA = 0 #disutility of not being in St Andrews

Num_UG= 8260 # Number of undegrards

Num_PG = 2164 # Number of postgrads

total_students = Num_UG+Num_PG

Prop_UG = Num_UG/total_students

Prop_PG = Num_PG/total_students

#NOTE: UGL - undergrad low income, PGL - postgrad low income and so
on

# this naming convention is followed throughout the code


Quantity = 7156 # number of available rooms

e = math.e  # initialising eulers number


def PSA(utility,sig,R): # return probability of being in STA based
on the sigmoid function

  return  (1-1 / (1 + e**((utility-Disutility_notSTA-R)/sig)))

  #implicit assumption made here that the error is logistically
distributed


def demand_UGL(Prop_UGL,WTP_UGL,sig,R):   # calculates demand for
Undergrad Low income

    UGL = Prop_UGL*Num_UG*(PSA(WTP_UGL,sig,R))

    return UGL

def demand_UGH(Prop_UGL,WTP_UGH,sig,R):    # calculates demand for
Undergrad High income

    UGH = (1-Prop_UGL)*Num_UG*(PSA(WTP_UGH,sig,R))

    return UGH
```

```
def demand_PGL(Prop_PGL,WTP_PGL,sig,R):  # calculates demand for Postgrad Low income

    PGL = Prop_PGL*Num_PG*(PSA(WTP_PGL,sig,R))

    return PGL

def demand_PGH(Prop_PGL,WTP_PGH,sig,R):  # calculates demand for Postgrad high income

    PGH = (1-Prop_PGL)*Num_PG*(PSA(WTP_PGH,sig,R))

    return PGH

def constraint_UG(parameters):    #inequality

    (Prop_UGL,Prop_PGL, WTP_UGL, WTP_UGH, WTP_PGL, WTP_PGH, R,sig) = parameters

    return WTP_UGH-WTP_UGL


def constraint_PG(parameters):   #inequality

    (Prop_UGL,Prop_PGL, WTP_UGL, WTP_UGH, WTP_PGL, WTP_PGH, R,sig) = parameters

    return WTP_PGH-WTP_PGL


def objective(parameters): # returns all the relevant theoretical moments and parameters to fsolve

    (Prop_UGL,Prop_PGL, WTP_UGL, WTP_UGH, WTP_PGL, WTP_PGH, R,sig) = parameters


    total_demand = demand_UGL(Prop_UGL,WTP_UGL,sig,R)+demand_UGH(Prop_UGL,WTP_UGH,sig,R)+demand_PGL(Prop_PGL,WTP_PGL,sig,R)+demand_PGH(Prop_PGL,WTP_PGH,sig,R)

    total_demand_2 = demand_UGL(Prop_UGL,WTP_UGL,sig,R+1)+demand_UGH(Prop_UGL,WTP_UGH,sig,R+1)+demand_PGL(Prop_PGL,WTP_PGL,sig,R+1)+demand_PGH(Prop_PGL,WTP_PGH,sig,R+1)


    Proportion_UGL = demand_UGL(Prop_UGL,WTP_UGL,sig,R)/(Prop_UGL*Num_UG) - 0.6

    Proportion_UGH = demand_UGH(Prop_UGL,WTP_UGH,sig,R)/((1-Prop_UGL)*Num_UG) - 0.8

    Proportion_PGL = demand_PGL(Prop_PGL,WTP_UGH,sig,R)/(Prop_PGL*Num_PG) - 0.5

    Proportion_PGH = demand_PGH(Prop_PGL,WTP_PGH,sig,R)/((1-Prop_PGL)*Num_PG) - 0.7
```

```
    Rent = R - 580

    Market_clearing = total_demand-Quantity   # remove quantity as
it's not independant

    elasticity_D = (total_demand_2-total_demand)*R/(total_demand) +
0.76

P_c = 1 # constant for proportion

E_c = 1 # constant for elasticity

M_c = 1 # constant for market clearing

R_c = 1 # constant for Rent

error_sum = P_c*((Prop_UGL-0.5)**2 + (Prop_PGL-0.5)**2)

list_of_equations =
[Proportion_UGL,Proportion_UGH,Proportion_PGL,Proportion_PGH,M_c*Mar
ket_clearing/7156**0.5,E_c*elasticity_D, R_c*Rent/580**0.5]

    for i in list_of_equations:

      error_sum = error_sum + i**2

    return error_sum# returning the list of equations

prop_limit = (0.02,0.98)

WTP_limit = (300,1500)

rent_limit = (350,850)

sig_limit = (2,1000)


limits =
(prop_limit,prop_limit,WTP_limit,WTP_limit,WTP_limit,WTP_limit,rent_
limit,sig_limit)


con1 = {'type':'ineq','fun':constraint_UG}

con2 = {'type':'ineq','fun':constraint_PG}

Guess = [0.1,0.2,500,550,525,575,20,500] #Prob_UGL,Prob_PGL,
WTP_UGL, WTP_UGH,WTP_PGL,WTP_PGH, R, sig


Results = minimize(objective,Guess,method='SLSQP',
bounds=limits,constraints=cons)

Results


#(Prop_UGL,Prop_PGL, WTP_UGL, WTP_UGH, WTP_PGL, WTP_PGH, R,sig)

def total_demand(Prop_UGL,Prop_PGL, WTP_UGL, WTP_UGH, WTP_PGL,
WTP_PGH, R,sig):
```

```
    total =
(demand_UGL(Prop_UGL,WTP_UGL,sig,R)+demand_UGH(Prop_UGL,WTP_UGH,sig,
R)+demand_PGL(Prop_PGL,WTP_PGL,sig,R)+demand_PGH(Prop_PGL,WTP_PGH,si
g,R))

    return total


end_UGL = demand_UGL(Results.x[0], Results.x[2], Results.x[7],
Results.x[6])

end_UGH = demand_UGH(Results.x[0], Results.x[3], Results.x[7],
Results.x[6])

end_PGL = demand_PGL(Results.x[1], Results.x[4], Results.x[7],
Results.x[6])

end_PGH = demand_PGH(Results.x[1], Results.x[5], Results.x[7],
Results.x[6])

a = total_demand(Results.x[0], Results.x[1], Results.x[2],
Results.x[3], Results.x[4], Results.x[5], Results.x[6],
Results.x[7])

b = total_demand(Results.x[0], Results.x[1], Results.x[2],
Results.x[3], Results.x[4], Results.x[5], Results.x[6]+1,
Results.x[7])

elasticity_D = ((b-a)*Results.x[6])/a


print(elasticity_D)

print(a)


##outputs


x = {'UGL': [Results.x[0], np.round(end_UGL/(Num_UG*Results.x[0]),
decimals=4), np.round(end_UGL/a, decimals=4), np.round(Results.x[2],
decimals=2), np.round(end_UGL, decimals=0), 'N/A'],

     'UGH': [(1-Results.x[0]), (np.round(end_UGH/(Num_UG*(1-
Results.x[0])), decimals=4)), np.round(end_UGH/a, decimals=4),
np.round(Results.x[3], decimals=2), np.round(end_UGH, decimals=0),
'N/A'],

     'PGL': [Results.x[1], np.round(end_PGL/(Num_PG*Results.x[1]),
decimals=4), np.round(end_PGL/a, decimals=4), np.round(Results.x[4],
decimals=2), np.round(end_PGL, decimals=0), 'N/A'],

     'PGH': [(1-Results.x[1]), (np.round(end_PGH/(Num_PG*(1-
Results.x[1])), decimals=4)), np.round(end_PGH/a, decimals=4),
np.round(Results.x[5], decimals=2), np.round(end_PGH, decimals=0),
'N/A'],

     'Total':['N/A', 'N/A', 'N/A', np.round(Results.x[6],
decimals=2), np.round(a), elasticity_D]}
```

```
index1 = ['Proportion of Students Who are Low/High Income',
'Proportion of Students Living in St. Andrews for Each Bin',
'Proportion of St Andrews Rooms Let to Each Bin (Demand of Bin/Total
Rooms)', 'Willingness to Pay for each Bin & Equilibrium Rent',
'Number of Students from each Bin Living in St Andrews', 'Elasticity
of Demand']


df = pd.DataFrame(data=x, index=index1)

df


##number of students from each bin living out of st andrews =
outend_bin

outend_UGL = (Num_UG*Results.x[0]) - end_UGL

outend_UGH = (Num_UG*(1-Results.x[0])) - end_UGH

outend_PGL = (Num_PG*Results.x[1]) - end_PGL

outend_PGH = (Num_PG*(1-Results.x[1])) - end_PGH


print([outend_UGL, outend_UGH, outend_PGL, outend_PGH])
```

Appendix 2

https://docs.google.com/spreadsheets/d/1u4i6zLOzmi7WZKhIF4qbUqX4r93wNIP-boiKg24-zN0/edit?usp=sharing